Module - VI

Dimensional Analysis.

It is a method for finding the relation b/w properties by using their dimensions. There are 2 types of units ;

-) Fundamental unit (M, L, T) and
-) Derived units

These are mainly 3 types of properties

1) Geometrical property - which gives the indications of size, shape and other parameters.

   Eg: length, breadth, height, area, Volume

2) Flow property - which indicates the motion parameters of fluid

   Eg: Velocity, acceleration, speed etc.

3) Fluid property - which indicates the fluid parameters which may or may not change due to the application of external agencies. (Force, temperature etc)

Eg: ~~Visa~~ Kinematic Viscosity, dynamic viscosity, Density etc .

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Dimensional homogeneity

It means the dimensions of each term in an equation on both side are equal.

This types of equations are called homogeneous equations.

Eg: $F = ma$

- Dimensional formulae of some properties

- Length, Breadth, Height $= L$
- Area $= L^2$
- Volume $= L^3$
- Force, $F = ma$

$= kg \frac{m}{s^2}$

$= L \frac{M}{T^2} = MLT^{-2}$

- $F = MLT^{-2}$

- Velocity, $V = \frac{\text{Displacement}}{\text{Time}} = \frac{L}{T}$

- $V = LT^{-1}$

•) Acceleration, $a = \frac{\text{Velocity}}{\text{Time}} = \frac{LT^{-1}}{T} = LT^{-2}$

•) $a = LT^{-2}$

•) Density, $\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{Kg}{L^3}$

•) $\rho = ML^{-3}$

•) Dynamic viscosity, $\mu = \frac{\tau\, dy}{du}$

Since $\tau = \mu \frac{du}{dy}$

$\mu = \frac{Kg\ \cancel{ms}}{s^2 m\ \cancel{m}}$

$\tau = \frac{\text{Force}}{\text{Area}}$

dy - distance/length $= \frac{N}{m^2}$

du - velocity $= \frac{Kg\, \cancel{m}}{s^2 m^{\cancel{2}}}$

$= \frac{Kg}{s^2 m}$

$\mu = \frac{kg}{s\, m}$

$\mu\ ML^{-1}T^{-1}$

$\mu = \frac{M}{TL}$

•) $\mu = MT^{-1}L^{-1}$

•) Kinematic viscosity, $\upsilon = \frac{\mu}{\rho}$

$= \frac{\cancel{M}T^{-1}L^{-1}}{\cancel{M}L^{-3}} = T^{-1}L^{2}$

MT^{2}

•) $\upsilon = TL^{2}$

1) <u>Rayleigh method</u> : It is used to determine the expression of a variable which depends on maximum 3 or 4 variable only.

1 Q) find the expression for power P developed by a pump when it depends upon Head - H , discharge - Q and specific weight - w of the liquid.

$$P \propto H Q w$$

$$P = \frac{work}{time} = \frac{NM}{S} = \frac{kgM}{S^2}\frac{M}{S}$$

$$= \frac{kg\, M^2}{S^3} = M L^2 T^{-3}$$

$$P = K H^a Q^b w^c \qquad Q = \frac{M^3}{S} = L^3 T^{-1}$$

$$M L^2 T^{-3} = K (L)^a (L^3 T^{-1})^b (M L^{-2} T^{-2})^c$$

Powers of M => $1 = c$

Powers of L => $2 = a + 3b - 2c$

$2 = a + 3 - 2$

$a = 1$

Powers of T => $-3 = -b - 2c$

$3 = b + 2$

$b = 1$

$P = K H Q w$

when $K = 1$

$P = \underline{\underline{\rho g Q H}}$.

work

Q) find the expression for drag force on smooth sphere of diameter D moving with a uniform velocity 'V' in a fluid density 'ρ' & dynamic viscosity μ.

$$F \propto D^a V^b \rho^c \mu^d$$

$$F = K D^a V^b \rho^c \mu^d$$

$F = ma$

$= Kg \frac{m}{s^2}$

$= M L T^{-2}$

D = L for

$$M L T^{-2} = K L^a (L T^{-1})^b (M L^{-3})^c (M T^{-1} L^{-1})^d$$

$D = L$

$V = \frac{dL}{T} = L T^{-1}$

$\rho = \frac{m}{V} = \frac{Kg}{m^3} = M L^{-3}$

$\mu = M T^{-1} L^{-1}$

Powers of M, 1 = c + d

$\underline{c = 1 - d}$

" L 1 = a + b − 3c − d

1 = a + 2 − d − 3(1 − d) − d

1 = a + 2 − d − 3 + 3d − d

$-2d + 3d$

" " T −2 = −b − d

2 = b + d

b = 2 − d

1 = a − 1 + d

2 = a + d

a = 2 − d

$$F = K D^{(2-d)} V^{(2-d)} \rho^{(1-d)} \mu^{d}$$

$$= K \frac{D^2}{D^d} \frac{V^2}{V^d} \frac{\rho}{\rho^d} \mu^d$$

$$F = \frac{K D^2 V^2 \rho \mu^d}{D^d V^d \rho^d}$$

$$F = K D^2 V^2 \rho \, f\left(\frac{\mu}{D V \rho}\right)^d$$

⇒ Bucking ham's π theorem

It is very difficult to find the relation if the variables are more than fundamental dimensions (M, L, T), in case of Rayleigh method this can be overcome by using Bucking Ham's π theorem.

It ~~states~~ that if there are 'n' variables (dependent & independent) in a physical problem & if this variables contains 'm' fundamental dimensions (M, L, T). Then the variables are arranged into '$n-m$' dimensionless terms which are called π terms.

Q) The resisting force 'R' of a supersonic plane during flight can be considered as, depends upon the length of air craft 'l', flow velocity V, air viscosity μ, air density ρ & Bulk Modulus of air 'K'. Express the functional relationship b/w these variables & resisting force.

$R \propto K l V \mu \rho$

$R = f(l, V, \mu, \rho, K)$

$0 = f_1(R, l, V, \mu, \rho, K)$

$n = 6 \quad m = 3$

No: of π terms $= n - m$
$= 6 - 3 = \underline{\underline{3}}$

$0 = f_1(\pi_1, \pi_2, \pi_3)$

Each π term contains $\Rightarrow$ (No: of π-terms + 1) variables

$R \propto K l V \mu \rho K$

$R = f(l, V, \mu, \rho, K)$.

$0 = f_1(R, l, V, \rho K)$.

Ans: By selecting the repeating variables, follow these considerations

Geometric property →

i) The dependent variable shouldn't be selected as repeating variable.

ii) The repeating variable should be chosen in such a way that one variable contain geometrical property (l, b, h etc),

one variable contain flow property (Velocity, acceleration, speed etc)

$ the 3rd variable contain fluid property (μ, ρ, ν etc)

$F = ma$
$= MLT^{-2}$

$\therefore \pi_1 = l^{a_1} V^{b_1} \rho^{c_1} R$

$\pi_2 = l^{a_2} V^{b_2} \rho^{c_2} \mu$

$\pi_3 = l^{a_3} V^{b_3} \rho^{c_3} K$

Consider π_1

$$M^0 L^0 T^0 = (L)^{a_1} (LT^{-1})^{b_1} (ML^{-3})^{c_1} (MLT^{-2})$$

$M \Rightarrow 0 = c_1 + 1$
$\Rightarrow c_1 = -1$

$L \Rightarrow 0 = a_1 + b_1 - 3c_1 + 1 \Rightarrow a_1 - 2 + 3 + 1 = 0$
$a_1 = -2$

$T \Rightarrow 0 = -b_1 - 2 \Rightarrow b_1 = -2$

$\cancel{\pi_1 = \rho (L)^{-2} (LT^{-1})^{-2} (ML^{-3})^{-1} (MLT^{-2})}$

$\pi_1 = l^{-2} V^{-2} \rho^{-1} R$

$$\pi_1 = \frac{R}{\rho L^2 V^2}$$

Consider π_2.

$M^0 L^0 T^0 = (L)^{a_2} (LT^{-1})^{b_2} (ML^{-3})^{c_1} (MT^{-1}L^{-1})$

$M \Rightarrow 0 = c_1 + 1$

$\Rightarrow c_2 = -1$

$L \Rightarrow 0 = a_2 + b_2 - 3c_1 - 1$

$a_2 - \cancel{2} + 3 - 1 = 0$

$a_2 = -1$

$T \Rightarrow 0 = -b_2 - 1$

$b_2 = -1$

$\pi_2 = l^{-1} V^{-1} \rho^{-1} \mu$

$$\pi_2 = \frac{\mu}{l V \rho}$$

Consider π_3

$M^0 L^0 T^0 \Rightarrow l^{a_3} V^{b_3} \rho^{c_3} K$

Bulk modulus = direct stress / Volume strain

Stress

$$M^0 L^0 T^0 = L^{a_3} (LT^{-1})^{b_3} (ML^{-3})^{c_3} (ML^{-1}T^{-2})$$

$M \Rightarrow 0 = c_3 + 1$

$c_3 = \underline{-1}$

$L \Rightarrow 0 = a_3 + b_3 - 3c_3 - 1$

$a_3 - 2 + 3 - 1 = 0$

$\underline{a_3 = 0}$

$T \Rightarrow 0 = -b_3 - 2$

$b_3 = \underline{-2}$

Stress $= \frac{F}{a}$

$= \frac{N}{m^2}$

$= \frac{kg}{s^2} \frac{m}{m^2}$

$= ML^{-1}T^{-2}$

$$\pi_3 = l^0 V^{-2} \rho^{-1} K$$

$$\pi_3 = \frac{K}{\rho V^2}$$

$ML^{-1}T^{-2}$

$K \; ML^{-1}T^{-2} \; K, \; F/a$

$$\therefore f_1 (\pi_1, \pi_2, \pi_3) = 0$$

$$f_1 \left(\frac{R}{\rho l^2 V^2}, \frac{\mu}{l V \rho}, \frac{K}{\rho V^2} \right)$$

$$\frac{R}{\rho V^2 l^2} = f_2 \left(\frac{\mu}{\rho V l}, \frac{K}{\rho V^2} \right)$$

$$R = \rho V^2 l^2 f_2 \left(\frac{\mu}{\rho V l}, \frac{K}{\rho V^2} \right)$$

KTU Q) The pressure difference in a pipe of diam d, length l, due to turbulent flow depends on velocity v, viscosity μ, density ρ & roughness k. Obtain an expression for ΔP (change in pressure).

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pressure loss in short length

$$\Delta P = f(D, L, V, \mu, \rho, K)$$

$$f_1(\Delta P, D, L, V, \mu, \rho, K)$$

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$n = 7, \quad m = 3$

$n - m = 7 - 3 = \underline{4}$

$3 + 1 = \underline{4}$

$$f_1(\pi_1, \pi_2, \pi_3, \pi_4) = 0$$

$\pi_1 = D^{a_1}, V^{b_1}, \rho^{c_1}\ \Delta P$

$p = f(d, l, v, \mu, \rho, k)$

$f_1(d, l, v, \mu, \rho, k, p) = 0$

$\pi_2 = D^{a_2}\ V^{b_2}\ \rho^{c_2}\ l$

$f_1(\pi_1$

$\pi_3 = D^{a_3}\ V^{b_3}\ \rho^{c_3}\ \mu$

$\pi_4 = D^{a_4}\ V^{b_4}\ \rho^{c_4}\ K$

$K = L$

I st π term

$$M^0 L^0 T^0 = L^{a_1}(LT^{-1})^{b_1}(ML^{-3})^{c_1}(ML^{-1}T^{-2})$$

$M \Rightarrow 0 = c_1 + 1$

$c_1 = \underline{-1}$

$L \Rightarrow 0 = a_1 + b_1 - 3c_1 - 1$

$0 = a_1 - 2 + 3 - 1$

$a_1 = \underline{0}$

$T \Rightarrow 0 = -b_1 - 2$

$b_1 = \underline{-2}$

$$\pi_1 = D^0\ V^{-2}\ \rho^{-1}\ \Delta P = \frac{\Delta P}{\rho V^2}$$

2nd

$$M^0 L^0 T^0 \pi_2 = (L)^{a_2} (LT^{-1})^{b_2} (ML^{-3})^{c_2} \, L.$$

$M \Rightarrow 0 = c_2$

$c_2 = 0$

$L \Rightarrow 0 = a_2 + b_2 - 3c_2$

$= a_2 + 0 + 0 + 1$

$a_2 = \underline{\underline{-1}}.$

$T \Rightarrow 0 = -b_2$

$b_2 = 0$

$$\pi_2 = D^{-1} V^0 \rho^0 L$$

$$= \frac{L}{D}$$

4th

$$M^0 L^0 T^0 = L^{a_4} (LT^{-1})^{b_4} (ML^{-3})^{c_4} k$$

$M \Rightarrow 0 = c_4$

$L \Rightarrow 0 = a_4 + b_4 - 3c_4 + 1$

$0 = a_4 + 0 + 0 + 1$

$a_4 = -1$

$T \Rightarrow 0 = -b_4$

$b_4 = 0$

$$\pi_4 = D^{-1} V^0 \rho^0 k$$

$$= \frac{k}{D}$$

3rd

$$M^0 L^0 T^0 = L^{a_3} (LT^{-1})^{b_3} (ML^{-3})^{c_3} [ML^{-1}T^{-1}]$$

$M \Rightarrow 0 = c_3 + 1$

$c_3 = \underline{-1}$

$L \Rightarrow 0 = a_3 + b_3 - 3c_3 - 1$

$0 = a_3 - 1 + 3 - 1$

$a_3 = \underline{-1}$

$T \Rightarrow 0 = -b_3 - 1$

$b_3 = \underline{\underline{-1}}$

$\pi_3 = [D]^{-1} [V]^{-1} [\rho]^{-1} \mu$

$$\underline{\underline{\pi_3 = \frac{\mu}{DV\rho}}}$$

$\therefore f_1 (\pi_1, \pi_2, \pi_3, \pi_4) = 0$

$$f_1 \left(\frac{\Delta P}{\rho V^2}, \frac{l}{D}, \frac{\mu}{DV\rho}, \frac{K}{D} \right) = 0$$

$$\frac{\Delta P}{\rho V^2} = f_2 \left(\frac{l}{D}, \frac{\mu}{DV\rho}, \frac{K}{D} \right)$$

$$\underline{\underline{\Delta P = \rho V^2 \cdot f_2 \left(\frac{l}{D}, \frac{\mu}{DV\rho}, \frac{K}{D} \right)}}$$

Q) The frictional torque 'T' of a disc of diame D rotating at a speed 'N' in a spe. fluid of viscosity μ & density ρ in a turbulent flow is given by $T = D^5 N^2 \rho \, \phi\left(\frac{\mu}{D^2 N \rho}\right)$. Prove this by method dimensions.

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T → force × ⊥ dist

$T = f(D, N, \rho, \mu)$

$f_1(T, D, N, \rho, \mu) = 0$

$n = 5 \quad m = 3$

no. of π terms $= n - m = 5 - 3 = 2$, $f_1(\pi_1, \pi_2) = 0$

$\pi_1 = (D^{a_1}, N^{b_1}, \rho^{c_1}, T$

$\pi_2 = (D^{a_2}, N^{b_2}, \rho^{c_2}\ \mu$

$N = \text{rpm}$

$\pi_1 = \omega$

$= 1/T = \underline{\underline{T^{-1}}}$.

$T = Nm = \frac{kg\,m}{s^2} \cdot m$

1st

$M^0 L^0 T^0 = L^{a_1} (T^{-1})^{b_1} (ML^{-3})^{c_1}\ ML^2T^{-2}$

$= \underline{\underline{ML^2T^{-2}}}$

$M \Rightarrow 0 = c_1 + 1 \qquad L \Rightarrow 0 = a_1 - 3c_1 + 2$

$c_1 = \underline{\underline{-1}} \qquad 0 = a_1 + 3 + 2$

$a_1 = \underline{\underline{-5}}$

$F = ma$

$T = F \times r$

$= MLT^{-2}$

$T \Rightarrow 0 = -b_1 - 2$

$b_1 = \underline{\underline{-2}}$

$\pi_1 = D^{-5} N^{-2} \rho^{-1} T = \frac{T}{\rho N^2 D^5}$

2nd

$M^0 L^0 T^0 = L^{a_2} (T^{-1})^{b_2} (ML^{-3})^{c_2} (ML^{-1}T^{-1})$

$M \Rightarrow 0 = c_2 + 1 \qquad L \Rightarrow 0 = a_2 + -3c_3 - 1$

$c_2 = \underline{\underline{-1}} \qquad 0 = a_2 + 3 - 1$

$a_2 = \underline{\underline{-2}}$.

$T \Rightarrow 0 = -b_2 - 1$

$b_2 = \underline{-1}$

$$\pi_2 = D^{-2} N^{-1} \rho^{-1} \mu = \frac{\mu}{N \rho D^2}$$

$$f_1(\pi_1, \pi_2) = 0$$

$$f_1\left(\frac{T}{\rho N^2 D^5}, \frac{\mu}{N \rho D^2}\right) = 0$$

$$\frac{T}{\rho N^2 D^5} = f_2\left(\frac{\mu}{N \rho D^2}\right)$$

$$T = D^5 N^2 \rho \ \phi\left(\frac{\mu}{D^2 N \rho}\right)$$

Similitudes (Types of similarities)

For predicting the performance of hydraulic structures & hydraulic machines before actually it is manufactured. The model is a scaled replica of actual structure or machine. The actual structure or machine is called prototype. The study of models of actual machines is called model analysis. The merits of model analysis are

i) Performance of machine or structure can be easily predicted in advance from its model.

ii) The relation b/w various influencing factors by conducting test on the model.

iii) The merits of design can be predicted with the help of model testing.

The similarity b/w model & prototype is called <u>Similitude</u>. These are of 3 types :-

i) <u>Geometrical Similarity</u> :- The ratio of the dimensions in model & prototype are compared and when it is found to be equal, known as geometrical similarity. The ratio of parameters of prototype and model is called Scale ratio.

Eg: $\frac{L_p \text{ (length of prototype)}}{L_m \text{ length of model}} = L_r$

L_p / L_m, L_r

$$\frac{L_p}{L_m} = L_r$$

L_r - Scale ratio.

$$\frac{A_p}{A_m} = A_r \quad , \quad \frac{V_p}{V_m} = V_r$$

$$\frac{A_p}{A_m} = \frac{(L_p)^2}{(L_m)^2} = (L_r)^2$$

$$\frac{\forall_p}{\forall_m} = \frac{(L_p)^3}{(L_m)^3} = (L_r)^3$$

ii) <u>Kinematic similarity</u>: It means the similarity of motion b/w model & prototype.

Eg: $\frac{V_p}{V_m}$ $\frac{(\text{velocity of prototype})}{(\text{velocity of model})} = V_r$

V_r - scale ratio.

$\frac{V_p}{V_m} = V_r$

$\frac{a_p}{a_m}$ $\frac{(\text{acceleration of prototype})}{(\text{ '' '' model})} = a_r$

$\frac{a_p}{a_m}$.

iii) <u>Dynamic similarity</u>: It means the similarity of forces b/w model & prototype.

Eg: $\frac{(F_s)_p}{(F_s)_m}$ $\frac{\text{Surface tension force of prototype}}{\text{ '' '' '' '' model}}$

$\frac{(F_s)_p}{(F_s)_m} = (F_s)_r$

⇒ <u>Dimensionless Numbers</u>

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Types <u>of forces</u> in fluids:

The major forces acting on fluids are as follows :-

1) Inertia force (F_i) :

F_i = mass × acceleration

$= m \cdot a$ $\qquad\qquad \rho = \frac{mass}{vol}$

$= \rho \times vol \times \frac{V}{t}$ $\qquad\qquad a = \frac{V}{t}$

$= \rho \times V \times \frac{vol}{t}$ $\qquad\qquad \frac{m^3}{s} = Q = A \times V$

$= \rho \times V \times (A \times V)$

$= \rho A V^2$ $\qquad\qquad \because \frac{vol}{t} = Q = A \times V$

$F_i = \rho L^2 V^2$

$F_i = \rho L^2 V^2$

2) Viscous Force (F_v) :

We have $\tau = \frac{F_v}{A}$ $\qquad\qquad \tau = \frac{F_v}{A}$ $\quad F_G = m \cdot g$

$F_v = \tau \times A$ $\qquad\qquad F_v = \tau A$

$\tau = \mu \frac{du}{dy}$, $A = L^2$ $\qquad F_v = \mu V L$

$= \left(\mu \frac{du}{dy}\right) \cdot L^2$

$= \mu \times \frac{V}{\cancel{L}} \times L^{\cancel{2}}$

$F_v = \mu V L$ $\qquad\qquad \because \frac{du}{dy} = \frac{V}{L}$

3) Gravitational force (F_G) :

$F_G = m \cdot g$

$= \rho \times vol \times g$ $\qquad \left(\because \rho = \frac{m}{vol}\right.$

$= \rho \times L^3 \times g$ $\qquad \left.(\because vol = L^3\right)$

4) Pressure Force (F_P):

$$P = \frac{F_P}{A}$$

$$F_P = P \times A$$

$$F_P = P \times L^2 \quad (\because A = L^2)$$

5) Surface Tension Force (F_S)

$$\sigma = \frac{N}{M} = \frac{F_S}{L}$$

$$\sigma = \frac{N}{M} = \frac{F}{L}$$

$$F_S = \sigma \times L$$

6) Elastic force (F_e):

$$P = \frac{F_P}{A}$$
$$F_P = PL^2$$
$$\propto F_S$$

$$\text{Elastic Stress} = \frac{F_e}{A}$$

$$\sigma = \frac{F_S}{A}$$
$$F_S = \sigma A$$
$$K = \frac{F_e}{A}$$

$$F_e = K \cdot A$$

$$F_e = K \cdot L^2 \quad (\because A = L^2)$$

7) Reynold's Number:

ratio b/w Inertia force & viscous force.

$$Re = \frac{\text{Inertia force}}{\text{Viscous force}}$$

$$= \frac{\rho L^{\cancel{2}} V^{\cancel{2}}}{\mu \cancel{V} \cancel{L}} = \frac{\rho V L}{\mu}$$

$$\boxed{Re = \frac{\rho V L}{\mu}}$$

$$\boxed{Re = \frac{\rho V D}{\mu}}$$ → Pipe $\quad \frac{\mu}{\rho} = \nu$

$\frac{\mu}{\rho} = \nu$ = Kinematic viscosity

$$\boxed{Re = \frac{VD}{\nu}}$$

Froude's No: (Fe)

$\frac{\mu}{\rho} = \nu$

$$Fe = \sqrt{\frac{\text{Inertia force}}{\text{Gravitational force}}}$$

It is the ratio of square root of Inertia force to gravitational force

$$= \sqrt{\frac{\rho L^2 V^2}{\rho L^3 g}} \qquad \frac{V}{\sqrt{Lg}}$$

$$= \sqrt{\frac{V^2}{Lg}}$$

$$\boxed{Fe = \frac{V}{\sqrt{Lg}}}$$

Euler's No:

$$Eu = \sqrt{\frac{\text{Inertia force}}{\text{Pressure force}}}$$

$$= \sqrt{\frac{\rho L^2 \times V^2}{P \times L^2}} = \sqrt{\frac{\rho V^2}{P}}$$

$$Eu = \frac{V}{\sqrt{P/\rho}}$$

Weber's No:

$$Wb = \sqrt{\frac{\text{Inertia force}}{\text{Surface Tension force}}}$$

$$= \sqrt{\frac{\rho L^2 V^2}{\sigma \times L}} = \sqrt{\frac{\rho L V^2}{\sigma}}$$

$$= \frac{V}{\sqrt{(\sigma/\rho L)}}$$

Mach No: (M)

$$M = \sqrt{\frac{\text{Inertia force}}{\text{Elastic force}}} = \sqrt{\frac{\rho L^2 V^2}{K \cdot L^2}}$$

$$= \sqrt{\frac{\rho V^2}{K}}$$

$$\boxed{M = \frac{V}{\sqrt{K/\rho}}}$$

where $\sqrt{\frac{K}{\rho}} = C =$ vel of sound

$$\boxed{M = \frac{V}{C}}$$

$\frac{\text{Velocity of object}}{\text{" " sound}}$

M < 1 Subsonic

M = 1 Sonic

M > 1 Supersonic

$\frac{\rho VL}{\mu}$

8/4/19 Sunday

Model Laws

Reynold's Model Law :

We have Reynold's no: $Re = \frac{\rho V L}{\mu}$

According to model law,

Test ഉപയോഗിക്കുന്ന model

Variables of model = Variables of prototype.

$$\frac{\rho_m V_m L_m}{\mu_m} = \frac{\rho_p V_p L_p}{\mu_p}$$

Q) A pipe of diame 1.5m is required to transport an oil of specific gravity 0.9 and viscosity 3×10^{-2} poise at the rate of 3000 l/sec. Test were conducted on a 15 cm diam pipe using water at 20°C. find the velocity & rate of flow in the model. Viscosity of water at 20°C is 0.01 poise

poise or Ns/m^2

$D_m = 15$ cm $= 0.15$m

$D_p = 1.5$m

$S_p = 0.9$ kg/m³

$\mu_p = 3 \times 10^{-2}$ poise

$\mu_m = 0.01$ $\quad S_p = 0.9 = 0.9 \times 1000$
$= 900\ kg/m^3$

$S_m = 1000$

from Reynold's model law $\frac{S_m V_m L_m}{\mu_m} = \frac{S_p V_p L_p}{\mu_p}$

$$\frac{1000 \times V_m \times 0.15}{0.01} = \frac{900 \times V_p \times 1.5}{3 \times 10^{-2}}$$

$$\frac{V_p}{V_m} = \frac{1}{3}$$

$$\frac{V_p}{V_m} = 1/3$$

$Q_p = A_p \times V_p$ $\qquad Q_p = 3000\ l/s$

$V_p = \frac{Q_p}{A_p}$ $\qquad = 3\ m^3/s$

$$= \frac{3}{\frac{\pi}{4}(D_p)^2} = \frac{3}{\frac{\pi}{4} \times 1.5^2}$$

$$V_p = 1.697\ m/s$$

$$\frac{V_p}{V_m} = 1/3$$

$$1.697 \times 3 = V_m$$

$$V_m = 5.09\ m/s$$

$Q_m = A_m \times V_m$

$Q_m = \frac{\pi}{4} \times 0.15^2 \times 5.09 = 0.0899\ m^3/s$

Q) Water is flowing through a pipe of diam 30 cm at a velo of 4 m/s find the velocity of oil flowing in another pipe of diam 10 cm. If the condition of dynamic similarities satisfied b/w the 2 pipes. The viscosity of water & oil is 0.01 poise & 0.025 poise respectively. Take specific gravity of oil is 0.8.

ans:) $D_m = 10 \times 10^{-2}$ m $= \underline{0.1m}$ $\quad D_p = 30 \times 10^{-2}$ m $= 0.3m$

$V_p = 4$ m/s

$\mu_m = 0.025$ poise $\quad \mu_p = 0.01$ poise

$\rho_m = 0.8 \times 1000 \quad \rho_m = 0.8 \times 1000$

$\rho_m = 800$ kg/m^3

from Reynold's Law

$$\frac{\rho_m V_m D_m}{\mu_m} = \frac{\rho_p V_p D_p}{\mu_p}$$

$$\frac{\cancel{1000}\,800 \times V_m \times 0.1}{0.025} = \frac{\cancel{800}\,1000 \times 4m \times 0.3}{0.01}$$

$$V_m = 37.5 \text{ m/s}$$

Q) A ship 300 m long moves in sea water whose density is 1030 kg/m³. A 1:100 model of this ship is to be tested in a wind tunnel. The velocity of air in the wind tunnel around the model is 30 m/s & resistance of the model is 60 N. Determine the velocity of ship in sea water & also the resistance of ship in sea water. The density of air is given as 1.24 Kg/m³. Take kinematic viscosity of sea water & air as 0.012 stokes & 0.018 stokes respectively.

Dimensions of Model

$V_m = 30\,m/s$

$\rho_m = 1.24\,Kg/m^3$

$\nu_m = 0.018$ Stokes

Sideral $F_m = 60N$

$\frac{L_p}{L_m} = \frac{100}{1} = L_r$

$\frac{L_p}{L_m} = 100$

$L_p = 100$

$L_m = \frac{L_p}{100}$

$= \underline{\underline{3m}}$.

Dimensions of prototype

$L_p = 300m$

$\rho_p = 1030\,Kg/m^3$

$\nu_p = 0.012$ stoke

$F_p = ?$

$\nu_m = \frac{\mu_m}{\rho_m}$

$\mu_m = 0.018 \times 1.24$

$= 0.022$

$\nu_p = \frac{\mu_p}{\rho_p} =$

$\mu_p = 12.36$

$$\frac{\rho_m V_m L_m}{\mu_m} = \frac{\rho_p V_p L_p}{\mu_p}$$

$$\frac{V_m L_m}{\nu_m} = \frac{V_p L_p}{\nu_p}$$

$$\frac{V_p}{V_p} = \frac{\cancel{0.0148}\ \cancel{L_m} \times 3}{0.018} = \frac{V_p \times 300}{0.012}$$

$$\frac{30 \times 3}{0.018} = \frac{V_p \times 300}{0.012} = \underline{\underline{0.2}}$$

$$\frac{F_p}{F_m} = \frac{\rho_p\, \cancel{A} L_p^2 V_p^2}{\rho_m\, L_m^2 V_m^2}$$

$F = ma$

$= \rho \times \text{Vol} \times \frac{V}{t}$

$= \rho \times A V^2$

$= \rho L^2 V^2$

$\frac{\text{Vol}}{t} = \frac{m^3}{s}$

$= Q$

$= A \times V$

$$\frac{F_p}{60} = \frac{1030 \times 300^2 \times 0.2^2}{1.24 \times 3^2 \times 30^2}$$

$$F_p = 22150.53\,N$$

VQ

KTU

Froude's law

we have

froude's No:, $F_e = \frac{V}{\sqrt{hg}}$.

$\therefore$ Model law

$$\frac{V_m}{\sqrt{h_m g_m}} = \frac{V_p}{\sqrt{h_p g_p}}$$

but $g_m = g_p$

$$\frac{V_m}{\sqrt{h_m}} = \frac{V_p}{\sqrt{h_p}}$$

$$\frac{V_p}{V_m} = \frac{\sqrt{h_p}}{\sqrt{h_m}}$$

$$\boxed{\frac{V_p}{V_m} = \sqrt{h_r}}$$

Scale ratio for various physical quantities.

1) Scale ratio for time :

We have velocity $V = \frac{L}{T}$

$\therefore T = \frac{L}{V}$

$T_r = \frac{L_p}{L_m} =$

$$T_r = \frac{T_p}{T_m} = \frac{\left(\frac{L_p}{V_p}\right)}{\left(\frac{L_m}{V_m}\right)}$$

$$= \frac{L_p}{V_p} \times \frac{V_m}{L_m} = \frac{L_p}{L_m} \times \frac{V_m}{V_p}$$

$$= L_r \times \frac{1}{\sqrt{L_r}} = \sqrt{L_r}$$

$$\boxed{T_r = \sqrt{L_r}}$$

2) Scale ratio for acceleration

$$a = \frac{V}{T}$$

$$a_r = \frac{a_p}{a_m} = \frac{\left(\frac{V_p}{T_p}\right)}{\left(\frac{V_m}{T_m}\right)} = \frac{V_p}{T_p} \times \frac{T_m}{V_m}$$

$$= \frac{V_p}{V_m} \times \frac{T_m}{T_p} = \sqrt{L_r} \times \frac{1}{\sqrt{L_r}} = 1$$

$$\boxed{a_r = 1}$$

3) Scale ratio ~~for~~ discharge:

$$Q = A \times V$$

$$Q = L^2 \times V$$

$$= \frac{Lp^2}{Lm^2} \times \frac{Vp}{Vm}$$

$$= (Lr)^2 \times \sqrt{Lr}$$

$$= \underline{\underline{Lr^{3/2}}}$$

4) Scale ratio for force

$$F = ma \qquad \rho = \frac{m}{V}$$

$$F = \rho L^2 V^2 \qquad m = \rho V$$

$$Fr = \frac{Fp}{Fm} = \frac{\rho p \, Lp^2 \, Vp^2}{\rho m \, Lm^2 \, Vm^2} \qquad a = \frac{V}{t}$$

$$Fr = \qquad \frac{\rho p}{\rho m} = 1$$

when density of model & prototype is same then $\rho r = \frac{\rho p}{\rho m} = 1$

$$Fr = \frac{Fp}{Fm} = \frac{\cancel{\rho p} \, Lp^2 \, Vp^2}{\cancel{\rho m} \, Lm^2 \, Vm^2}$$

$$= \frac{Lp^2}{Lm^2} \times \frac{Vp^2}{Vm^2} = (Lr)^2 \times (\sqrt{Lr})^2$$

$$= Lr^{3}$$

$$\boxed{Fr = Lr^3}$$

5) Scale ratio for pressure P_r :

$$P = \frac{F}{A}$$

$$P_r = \frac{L_r^3}{L_r^2} = 1 \; F$$

$$P_r = \frac{P_p}{P_m} = \frac{\left(\frac{F_p}{A_p}\right)}{\left(\frac{F_m}{A_m}\right)} = \frac{F_p}{A_p} \times \frac{A_m}{F_m}$$

$$= \frac{F_p}{L_p^2} \times \frac{L_m^2}{F_m} = L_p^2$$

$$= \frac{F_p}{F_m} \times \frac{L_m^2}{L_p^2} = L_r^3 \times \frac{1}{(L_r)^2}$$

$$\boxed{P = L_r}$$

6) Scale ratio for work (Moment, torque etc)

$W = F \times d \quad W = F \times l$

$$W_r = \frac{W_p}{W_m} = \frac{F_p}{F_m} \times \frac{l_p}{l_m}$$

$$= F_r \times L_r$$

$$= L_r^3 \times L_r = \underline{L_r^4}$$

7) Scale ratio for power

$$P = \frac{\text{Work}}{\text{Time}} = \frac{\left(\frac{W_p}{W_m}\right)}{\left(\frac{T_p}{T_m}\right)} = \frac{W_p}{W_m} \times \frac{T_m}{T_p}$$

$$= W_r \times \frac{1}{T_r} = L_r^4 \times \frac{1}{\sqrt{L_r}} = \underline{L_r^{7/2}}$$

Q) In a 1:40 model of a spill way, the velocity & discharge are 2 m/s & 2.5 m³/s. find the corresponding velo & discharge of prototype.

$$L_r = \frac{L_p}{L_m} = \frac{40}{1} = \underline{40}$$

$$\frac{V_p}{V_m} = \sqrt{L_r}$$

$$\frac{V_p}{2} = \sqrt{40} \Rightarrow V_p = \underline{12.6} \text{ m/s}$$

$$Q_m = 2.5 \text{ m}^3/\text{s}$$

$$\frac{Q_p}{Q_m} = Q_r = L_r^{3/2}$$

$$Q_p = 40^{3/2} \times 2.5$$

$$Q_p = \underline{632.455} \text{ m}^3/\text{s}$$

Types of models

Hydraulic models are classified as

i) Undistorted Models : These are models which are geometrically similar to their prototype or if the scale ratio for the model & prototype is same. The behaviour of prototype can be easily predicted from this type of models.

9P) Distorted Models: ~~These~~ A model is said to be distorted if it is not geometrically similar to its prototype. for this type of models various scale ratios are adopted. for eg: In case of river 2 diff scale ratios are adopted for the model analysis; one for the width of the river & another for the depth of the river.